statistical problem is solved by a combination of analysis and computer simulation with pseudo-random numbers.

The book surveys applications to pure mathematics, to nuclear physics, to statistical mechanics, to mathematical statistics, and to theoretical chemistry. Many original results are presented. An original discussion is presented of percolation processes, which involve deterministic flows in random media.

Through many examples the book stresses that the most obvious formulation of a Monte Carlo problem is not always the best. The required answer is a statistical expected value. To attain a given accuracy with a specified probability, the number of times an unbiased statistic must be computed is proportional to the variance of the statistic. Different Monte Carlo formulations, all of which yield the same expected value, are shown to produce very different variances.

The chapters entitled "Conditional Monte Carlo" and "Percolation Processes" are particularly fascinating. In problems involving conditional probabilities, the proper embedding in a larger measure space may make the difference between practical solvability and unsolvability. In certain complex statistical problems, such as percolation processes, direct simulation is impractical. But these problems may be shown analytically to be equivalent to other statistical problems which are solvable by direct simulation with presently available computers.

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46[L].—A. R. Curtis, *Coulomb Wave Functions*, Royal Society Mathematical Tables, Volume 11, Cambridge University Press, New York, 1964, xxxv + 209 pp., 28 cm. Price $15.00.

Schrödinger's equation for a hydrogen-like atom or ion in a potential field takes the form

$$\frac{h^2}{2\mu}\frac{d^2R}{dr^2} + \left\{W + \frac{Ze^2}{r} - \frac{h^2}{2\mu}\frac{L(L+1)}{r^2}\right\} R = 0,$$

on separating in polar co-ordinates. Here $-e$ denotes the charge and W the total energy of the electron, and Ze denotes the nuclear charge. The most extensive existing tables of solutions of this differential equation are those of the National Bureau of Standards [1], [2]. In these tables the independent variable used is $(2\mu W)^{1/2} h^{-1} r$. In the present tables the independent variable is $x = \mu e^2 h^{-2} Zr$; with this choice, wave functions for different values of W can be compared directly for constant values of r, rather than for constant values of $rW^{1/2}$. Thus the standard form adopted for the differential equation is

$$\frac{d^2y}{dx^2} + \left\{a + \frac{2}{x} - \frac{L(L+1)}{x^2}\right\} y = 0,$$

in which x is positive, a ($= 2Wh^2Z^{-2}\mu^{-1}e^{-4}$) is real, and L is zero or a positive integer.

The tables cover the ranges $a = -2(0.2)2$; $L = 0, 1, 2$; $x = 0(0.1)10$ and $1/x = 0(0.002 \text{ or } 0.005)0.1$. The accuracy of the tabulated values is six decimals throughout, corresponding generally to six or seven significant figures in the actual solutions and their first x-derivatives. Modified second differences, and, where necessary, modified fourth differences, are provided for interpolation in the x-direction. The tables were reproduced photographically from copy prepared on a card-operated typewriter and the printing is quite clear.

One of the difficulties of this project, described in the Introduction to the tables, was the choice of standard solutions of the differential equation. In addition to the usual criteria for numerically satisfactory solutions of linear differential equations in exponential or oscillatory regions, a further physical desideratum is that interpolation should be feasible in the a-direction. This last requirement could be fulfilled only partially.

The Introduction also describes the computation of the tables at the National Physical Laboratory, and includes worked examples of x-wise and a-wise interpolation. Furthermore, many mathematical properties of the chosen solutions are derived, including expressions in terms of confluent hypergeometric functions and the N. B. S. Coulomb wave functions; recurrence relations; convergent expansions in series of powers of x and series of Bessel functions; asymptotic expansions for large x, large $|a|$, small $|a|$, and large L.

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1. NATIONAL BUREAU OF STANDARD, *Tables of Coulomb Wave Functions*, Vol. I, Applied Mathematics Series **17**, U. S. Government Printing Office, Washington, D.C., 1952.
2. NATIONAL BUREAU OF STANDARDS, *Handbook of Mathematical Functions*, Chapter 14, Applied Mathematics Series **55**, U. S. Government Printing Office, Washington, D.C., 1964.

47[L].—M. A. FISHERKELLER & W. J. CODY, *Tables of the Complete Elliptic Integrals K, K', E, and E'*, Argonne National Laboratory, Argonne, Ill., ms. of 14 typewritten pages deposited in the UMT File.

The authors tabulate K, E, K', and E' to 17S for $k = 0(0.005)1$ and for $k^2 = 0(0.005)1$. In a two-page introduction we are informed that the underlying computations were performed on a CDC-3600, using 25S, and the results were checked to at least 20S by means of Legendre's relation, before rounding to 17S. Accordingly, the tabulated values are believed to be accurate to within one-half a unit in the least significant figure.

Reference is made to some specialized tables of Airey [1] relating to values of K and E to 12 and 13D when k^2 approaches 1, and to the tables of Spenceley and Spenceley [2], wherein the argument is the modular angle. It seems appropriate to mention here that the WPA Project for the Computation of Mathematical Tables [3] in 1942 prepared manuscript tables of the Jacobi elliptic functions, which included as an auxiliary table values of K to 17S for $k^2 = 0(0.01)1$, that is, at twice the subinterval in k^2 appearing in these tables.

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1. J. R. AIREY, "Toroidal functions and complete elliptic integrals," *Philos. Mag.* (7), v. 19, 1935, pp. 177–188.
2. G. W. SPENCELEY & R. M. SPENCELEY, *Smithsonian Elliptic Functions Tables*, Smithsonian Institution, Washington, D. C., 1947. [See *MTAC*, v. 3, 1948/1949, pp. 89–92, RMT **485**.]
3. *MTAC*, v. 1, 1943/1945, pp. 125–126, UMT **12**.